# Screening contracts for information products in oligopoly

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**Abstract**

In this paper, I study the design of screening contracts for information products that are sold to a group of buyers who have strategic interactions with one another. An information provider offers a menu of information structures (i.e. experiments) to firms that compete in a downstream market. Firms can also obtain their own signals privately. The precision of own signals obtained is firm's private information (i.e. type). I first identify properties of a feasible menu under different strategic environments. A feasible menu will always provide as accurate or more accurate information to low types (i.e. low precision of own signals) of firms than to high types of firms. When firms face strategic complementarities, their expected net gain of the additional information is nonincreasing in types and, thus, high types may be excluded from the market. When firms face strategic substitutes, a feasible menu may exclude intermediate types. I then study how the nature of competition between firms affects the information provider's optimal menu. Compared to an environment with no strategic interactions, the presence of strategic complementarities leads the information provider to perfectly correlate the information structures and provide the most accurate information available. When firms face a game of strategic substitutes, the market may be partitioned into two segments and the information provided to the 'high type' segment is degraded.

*Keywords:* information market, mechanism design, adverse selection, strategic interactions

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# 1 Introduction

## 1.1 Motivation

As information technology develops, data about economic activity have been more easily collected, processed and traded than ever before. This has led to an emerging market in which business-to-business information services are provided. For example, firms such as Nielsen and IRI have built business models around collecting and selling information about consumers to other market participants such as advertisers and retailers.[1] It is widely accepted that providing more accurate information about consumers (or market demand) enhances downstream firms' production and marketing efficiency. However, in an oligopolistic market, the information provided also has an indirect effect due to the strategic interactions between firms. Furthermore, if there exists asymmetric information between firms and the information provider, it is even more tricky for the information provider to extract surplus from firms. How does the nature of competition in an oligopoly environment affect an information provider's strategy? What is the optimal selling mechanism for an information provider in the presence of adverse selection?

This paper studies these questions in a setting where downstream firms face uncertainty about the market demand intercept. Each firm can privately observe a signal and the precision of the signal is firm's private information (i.e. type). Besides that, there exists a monopolistic information provider that provides additional information to firms by offering a menu of information structures (i.e. experiments). Potential applications include firms, such as IRI and Nielsen, offering retail market analyses of varying precision to the consumer packaged goods industry, or financial data providers, such as Bloomberg and Thomson Reuters, offering expansive menus of information products.

To be more precise, I study a sequential game in which the information provider first offers a menu of information structures (possibly correlated with a constant correlation coefficient) and then downstream firms compete conditional on each firm choosing an option from the menu (according to their types and willingness to pay) and forming posterior beliefs about the true state and other firms' actions. In the presence of private information, I first apply the mechanism design approach to study the design of feasible menus. A menu is feasible if and only if it satisfies individual rationality (IR) constraints, incentive compatibility (IC) constraints and nonnegative payment (NP) constraints. By the revelation principle, an offered menu must be feasible in the equilibrium.

Conditional on imposing a feasible menu, the value of additional information is derived as a function

[1] See Bergemann and Bonatti (2019) for a survey on markets for information.

of firm's type, the precision of additional information from the provider and some parameters of the mechanism. I find that when firms have no strategic interactions, or face a game of strategic complementarities, all types of firms will have greater willingness to pay for additional information with higher precision. But when faced with a game of strategic substitutes, some types of firms may have greater willingness to pay for additional signals with lower precision.

This observation is due to the fact that there are two forces affecting firms' willingness to pay. One force is that receiving more accurate information aligns firms' actions with the true state. In any strategic environment, this force will lead firms to have greater willingness to pay for additional information with higher precision. The other force is that receiving more accurate information also aligns a firm's action with other firms' actions. This force has a positive effect when firms face a game of strategic complementarities and has a negative effect when firms face a game of strategic substitutes. Thus, in an environment of strategic complementarities, firms are even more amenable to paying for more accurate additional information. But with strategic substitutes, the willingness to pay is dampened since firms want their actions to be divergent with each other. What's more, when the second force dominates the first one, some types of firms even have greater willingness to pay for addtional signals with lower precision.

Based on this, I then identify some properties of feasible menus. A feasible menu will always provide as accurate or more accurate information to low types (i.e. low precision of own signals) than to high types. When firms have no strategic interactions or face a game of strategic complementarities, a feasible menu may exclude high types from the market. The intuition is that when firms face no strategic interactions, firms with low types have greater willingness to pay for additional information compared to high types. When firms face strategic complementarites, this effect is intensified since the correlated additional information now can act as a coordination device for firms. However, when firms face strategic substitutes, they want their actions more divergent with one another. Thus, receiving an additional signal has a negative effect on firms' expected profit, which weakens firm's willingness to pay for additional information. This leads to an entirely different feature for feasible menus in the environment of strategic substitutes. In this environment, a feasible menu may exclude intermediate types instead of high types. This is because some high types put extremely high weight on their own signals. Obtaining an additional signal that is possibily correlated with the one received by other firms does less harm to firms that put extremely low weight on this additional signal than to other types of firms. Thus compared to intermediate types, these high types of firms are more willing to buy additional signals.

Finally, I characterize the optimal menu under different strategic environments. I further study how the nature of competition between firms affects the information provider's optimal menu. As a benchmark, I characterize the optimal menu when firms face no strategic interactions. By assuming uniform distribution and relatively low variation in types, I make sure that all information structures are not distorted and no types are excluded from the market. This assumption allows us to isolate the effect of the nature of competition on the degree of informational distortion, and the choice of optimal menu by the information provider.

It shows that compared to an environment with no strategic interactions, firms facing strategic complementarities leads the information provider to perfectly correlate the information structures and provide the most accurate information available. In this sense, the information provider is essentially sending public signals. However, when firms face a game of strategic substitutes, the optimal menu will be such that either all signals are provided independently with the highest precision, or the market is segmented into two parts. For the first segment, firms have relatively low types (i.e. low precision of their own signals); the infomation provider offers the most accurate information to all types in this segment. For the second segment, firms have relatively high types and the information provided is distorted and degraded. This menu may generates higher expected profit than a single-item menu, under some conditions. By restricting the fraction of the 'low type' segment and providing degraded information to the 'high type' segment, the information provider enhances very low types' willingness to pay, whose surplus can be more heavily extracted.

### 1.2 Related literature

This paper contributes to a growing literature on information products and information markets. The paper that has a setting similar to us is Bimpikis et al. (2019). They show that the nature and intensity of competition among the information provider's customers play first-order roles in determining the information provider's optimal strategy. More specifically, they show that when the customers view their actions as strategic complements, the provider finds it optimal to offer the most accurate information at the provider's disposal to all potential customers. My paper, instead, studies an adverse selection problem in which firms hold private information about the precision to acquire information on their own. In the presence of adverse selection, the information provider finds it harder to subtract any surplus from firms. I also assume finite number of firms instead of a continuum of firms which is only plausible in a large oligopoly market. Admati and Pfleiderer (1986) analyzes selling information to a continuum of ex-ante homogeneous traders that all hold the same prior information. After the purchase of supplemental information, the agents trade an asset with a common value. They show that it is optimal to provide noisy, idiosyncratic and, hence, heterogeneous information. By contrast, I focus on heterogeneous types of buyers who value information differently, due to their different private information.

This paper is also related to a tranditional literature on information sharing in oligopoly. ( Novshek and Sonnenschein (1982), Clarke (1983), Vives (1984),Li (1985), Gal-Or (1985), Sakai and Yamato (1989).) In these models, the information was collected and shared by an intermediary, such as a trade association, that merely organized and facilitated the exchange between the oligopolists, but that had no financial incentive or market power. This earlier literature on information sharing leaves a limited role for information design. In particular, while the firms were allowed to add noise to their private information, the intermediary was restricted to simply aggregate and report the received information in the same format to all of the firms. My model explicitly considers the problem of an information provider and finds that the nature of competition between firms affects the information provider's choice of information quality.

Lastly, this paper also contributes to theory of information design and mechanism design. Bergemann et al. (2018) formalizes a model where a data seller sells supplemental data to a data buyer whose willingness to pay is determined by the quality of his initial private information. They provide an explicit construction of the optimal menu of experiments in the cases of binary states and actions, or binary types. I consider the design of the menu of experiments in a setting with multiple competitive buyers and continuous type and action space.

The structure of this paper is the following: Section 2 sets up the model, constructs the information provider's problem and the constraints for a feasible mechanism; Section 3 characterizes a generic firm's expected value from the information provider and then identifies several properties for feasible menus; Section 4 discusses the design of optimal menu; Section 5 concludes; Appendix includes all the proof.

## 2 Model

Consider a market with $n(\geq 2)$ firms competing in quantities and firm $i$ $(i = 1, 2, ..., n)$ faces a linear inverse demand function given by

$$p_i = \theta + dq_{-i} - q_i$$

where $q_{-i} = \frac{1}{n-1}\sum_{j\neq i} q_j$ is the average quantity of all firms except for firm $i$ and $d$ is an exogenous parameter measuring the degree of strategic complementarity in firms' actions. When $d < 0$, each firm's best response function is decreasing in other firms' actions and firms face a game of strategic substitutes. When $d > 0$, firms' actions are strategic complements. $d = 0$ refers to a market in which there is no strategic interactions. An alternative interpretation for the sign of $d$ can be that it measures whether firms' products are substitutes or complements. In our differentiated Cournot model, $d > 0$ means that they are complements. However, this is not a key factor affecting our conclusions since the model can be easily extended to a differentiated Bertrand model $q_i = \theta + dp_{-i} - p_i$ where $d > 0$ means that firms face a game of strategic complements but firms' products are substitutes. We further assume $d \in (-\infty, 1]$ to guarantee a unique interior equilibrium.

Firms have zero production cost and face uncertainty about the market demand intercept $\theta$. All firms hold a common prior belief on $\theta$, which, for simplicity, we assume to be a uniform improper prior distributed over the real line.[2]

Each firm privately and independently observes a signal for $\theta$. Firms may learn the true state through different sources, some signals generated are more informative and some contain more noisy information. Thus firms may receive signals with different quality/precision. For a generic firm, we assume the signal observed is

$$x = \theta + e \quad e \sim N(0, \sigma_x^2)$$

Denote $t = \frac{1}{\sigma_x^2}$. We assume that $t$ is a random variable and the realization of $t$ is firm's private information. Thus we can view it as firm's 'type'. For firm $i$, denote a generic type as $t_i$. It's common knowledge that $t_i, i = 1, ..., n$ are independently and identically drawn from the same distribution $F(.)$ with support $[\underline{t}, \bar{t}]$ where $0 < \underline{t} < \bar{t}$. We assume that $F(.)$ is strictly increasing.

There is a monopolistic information provider that who can costlessly learn the state $\theta$ and is disclosing the information about $\theta$ to firms by selling experiments (information structures). Since firms hold private information, optimally the information provider will screen firms by offering a menu of information structures. We denote the menu by $(\Sigma, m)$, where $\Sigma$ is a collection of information structures and $m$ is a payment function mapping from $\Sigma$ to $R^+$.

A generic element of $\Sigma$ is an information structure that can be represented by $I = (S, H)$, where $S$ denotes the set of signals and $H(.)$ is a distribution function mapping from $\Theta$ to $\Delta S$. We assume that,

[2]Our conclusions do not depend on the prior distribution of $\theta$ but assuming $\theta$ has a standard prior distriution with finite variance complicates calculations and propositions.

for $s \in S$ we have

$$s = \theta + \mu \ \mu \sim N(0, \sigma_s^2)$$

Denote the precision of signals as $\tau_s = \frac{1}{\sigma_s^2}$. We assume that there is an upper bound for the precision/quality of information provided, i.e., $\tau_s \in (0, \bar{\tau}]$.[3] Besides the offered menu, the information provider can also determine how different information structures are correlated with each other conditional on a realized state. Denote that, for any two information structures $I_a \in \Sigma$ and $I_b \in \Sigma$, the correlation coefficient between them conditional on $\theta$ is $\rho(I_a, I_b) = corr(\mu_a, \mu_b)$. We also assume that signals offered by the information provider are independent with each firm's own private signals conditional on the realized true state. In other words, $\mu$ is independent with $e$.

The timing of our game will be:

1. The seller offers a menu $M$ which is public information to all firms.

2. The true state of $\theta$ and each firm's type $t$ are realized.

3. Each firm privately chooses an information structure from the menu and pays a corresponding price.

4. Each firm's own private signal $x$ is realized and each firm privately observe a signal from the chosen information structure. Then firms compete in the downstream market.

Consider the information provider's problem. By the revelation principle, we can look at a direct mechanism $M = \{(a(t), m(t))_{t \in [\underline{t}, \bar{t}]}, (I(t))_{t \in \{k | a(k)=1\}}, \{\rho(I(t_i), I(t_j))\}_{t_i, t_j \in \{k | a(k)=1\}}\}$ where $a(t) \in \{0, 1\}$ is a binary allocation variable for type $t$ (1 means receiving an additional information structure and 0 means not), $I(t) = (S(t), H(t))$ is an information structure provided to type $t$ conditional on it actually receiving the information structure, $\rho(I(t_i), I(t_j))$ is the correlation coefficient between the information structures offered to type $t_i$ and type $t_j$ conditional on both receiving information structures and $m(t)$ is the transfer to type $t$.

For tractability, we assume that the information provider is imposing a constant nonnegative correlation coefficient $\rho \in [0, 1]$ among all pairs of information structures. Then we can represent a mechanism as $M = \{(a(t), m(t))_{t \in [\underline{t}, \bar{t}]}, (I(t))_{t \in \{t' | a(t')=1\}}, \rho\}$. Under our setting, an information structure is characterized by its precision. Thus we can also represent a mechanism as $M = \{(a(t), m(t))_{t \in [\underline{t}, \bar{t}]}, (\tau(t))_{t \in \{t' | a(t')=1\}}, \rho\}$.

Denote $\Pi^*(t, t', a(t'))$ as the expected profit of a firm with type $t$ when reporting $t'$. Denote $\Pi^*(t) = \Pi^*(t, t, a(t))$ and $\hat{\Pi}(t)$ as the expected profit of a firm with type $t$ not participating to the mechanism.[4]

[3]We assume that $\bar{\tau} > N\bar{t}$ for an arbitrary large number $N$. This assumption not only guarantees a large enough strategy space for the information provider but also avoids the intractability due to the unboundness.

[4]Rigorously speaking, the expected profit of a firm not participating to the mechanism should be conditional on the

Then a mechanism is individually rational (IR) if and only if

$$\Pi^*(t) - m(t) \geq \hat{\Pi}(t) \quad \forall t \in [\underline{t}, \bar{t}]$$

A mechanism is incentively compatible (IC) if and only if

$$\Pi^*(t) - m(t) \geq \Pi^*(t, t', a(t')) - m(t') \quad \forall t, t' \in [\underline{t}, \bar{t}]$$

A mechanism satisfies nonnegative payment (NP) constraints if and only if

$$m(t) \geq 0 \quad \forall t \in [\underline{t}, \bar{t}]$$

We could normalize these constraints by defining the expected value from the information provider to a firm with type $t$ when he is reporting $t'$ as $V(t, t')$. This is given by

$$V(t, t') = \Pi^*(t, t', a(t')) - \hat{\Pi}(t)$$

Denote $V(t) = V(t, t)$.

Then (IC) constraints become

$$V(t) - m(t) \geq V(t, t') - m(t') \quad \forall t, t' \in [\underline{t}, \bar{t}]$$

And (IR) constraints are

$$V(t) - m(t) \geq 0 \quad \forall t \in [\underline{t}, \bar{t}]$$

Then the information provider's problem can be represented by

$$\max_{\{(a(t), m(t))_{t \in [\underline{t}, \bar{t}]}, (\tau(t))_{t \in \{t' | a(t')=1\}}, \rho\}} E_{\boldsymbol{t}}\left(\sum_{i=1}^{n} m(t_i)\right) = n \int_{\underline{t}}^{\bar{t}} m(t) dF(t)$$

$$\begin{aligned} s.t. \quad & V(t) - m(t) \geq V(t, t') - m(t') && \forall t, t' \in [\underline{t}, \bar{t}] && (IC) \\ & V(t) - m(t) \geq 0 && \forall t \in [\underline{t}, \bar{t}] && (IR) \\ & m(t) \geq 0 && \forall t \in [\underline{t}, \bar{t}] && (NP) \end{aligned}$$

mechanisn since the firm's belief about other firms' types and actions is updated from the rejection. However, as shown in Lemma 1, the linear demand setting guarantees that there is no learning from rejection.

We say that a mechanism is feasible if and only if it satisfies (IC), (IR) and (NP).

## 3 Feasible menus

In this section, we identify the properties that any feasible menu should satisfy. Suppose a feasible mechanism is implemented, the following lemma characterizes the equilibrium strategy of a firm in the competition stage as a function of its type, observed own signal and the signal from the information provider.

**Lemma 1.** *Suppose the mechanism $M = \{(a(t), m(t))_{t\in[\underline{t},\bar{t}]}, (\tau(t))_{t\in\{t'|a(t')=1\}}, \rho\}$ imposed by the information provider is feasible. If firm $i$ with type $t_i$ receives additional information from the provider ($a(t_i) = 1$), then its equilibrium strategy is given by*

$$q_i^*(x_i, s_i, t_i) = \frac{1}{2-d}[k(t_i, d, \rho)x_i + (1 - k(t_i, d, \rho))s_i]$$

*where $k(t_i, d, \rho) = A(d, \rho)r(t_i)$ , $A(d, \rho) = \frac{2-d\rho\int_{\{t|a(t)=1\}} dF(t)}{2-d\rho\int_{\{t|a(t)=1\}} \frac{t}{t+\tau(t)}dF(t)}$, $r(t_i) = \frac{t_i}{t_i+\tau(t_i)}$, $x_i$ is observed own signal and $s_i$ is observed signal from the information provider. If firm $i$ with type $t_i$ does not receive additional information from the provider ($a(t_i) = 0$), then its equilibrium strategy is given by*

$$q_i^*(x_i, t_i) = \frac{1}{2-d}x_i$$

*where $x_i$ is observed own signal.*

In Lemma 1, notice that if a firm with type $t_i$ receives additional information from the provider ($a(t_i) = 1$), $k(t_i, d, \rho)$ measures the weight this firm's equilibrium strategy puts on its own signal. Fix the type $t_i$, $k(t_i, d, \rho)$ is decreasing in $\tau(t_i)$. Intuitively, if a specific type firm receives a signal with higher precision from the information provider, its equilibrium strategy will put a higher weight on $s_i$. What's more, $k(t_i, d, \rho)$ is the product of two components: $r(t_i)$ and $A(d, \rho)$. $r(t_i)$, which is a 'private component', measures the relative precision of a firm's own signal ($\frac{t_i}{t_i+\tau(t_i)}$). $A(d, \rho)$, which is a 'public component', does not depends on firm's own type but depends on the degree of strategic complementarity ($d$), the correlation coefficient between signals offered by the information provider ($\rho$), the fraction of types receiving additional signals ($\int_{\{t|a(t)=1\}} dF(t)$) and the expected relative precision over these types ($\int_{\{t|a(t)=1\}} \frac{t}{t+\tau(t)}dF(t)$). If firms face no strategic interactions ($d = 0$) or information structures offered by the provider are not correlated ($\rho = 0$), we have $A(d, \rho) = 1$ and $k(t, d, \rho) = r(t) = \frac{t}{t+\tau(t)}$. Thus the weight is solely a 'private component' $\frac{t}{t+\tau(t)}$ which is the firm's relative precision of own signal. If firms face strategic interactions ($d \neq 0$) and information structures are positively correlated ($\rho > 0$), we have $A(d, \rho) < 1$ if $d > 0$ and $A(d, \rho) > 1$ if $d < 0$. When $d < 0$ and $A(d, \rho) > 1$, the weight $k(t, d, \rho)$ is adjusted upward compared to the case when $A(d, \rho) = 1$. Thus firm's equilibrium strategy put a higher weight on its own signal. When $d > 0$ and $A(d, \rho) < 1$, the weight $k(t, d, \rho)$ is adjusted downward compared to the case when $A(d, \rho) = 1$. Thus firm's equilibrium strategy puts a lower weight on its own signal.

Another remark for Lemma 1 is that, the equilibrium strategy a firm with type $t_i$ who does not receive any additional information ($a(t_i) = 0$) are the same as those of a firm having the same type but not participating to the mechanism, i.e., $q^*(x_i, t_i) = \hat{q}(x_i, t_i)$. This is because in both scenarios firms with the same type have the same information set thus have the same equilibrium outcome. What's more, if we think of an incomplete information game in which the information provider does not exist, the equilibrium strategy a firm with type $t_i$ who observes $x_i$ is also the same as $q^*(x_i, t_i)$. In other words, if the firm chooses not to buy any additional information from the information provider, its equilibrium outcome is not affected by whether other firms buy any additional information or not. This implication depends crucially on the specification of linear demand. As implied by Lemma 1, firm $i$ knows that for another firm who receives additional information, such as firm $j$, its equilibrium strategy is a weighted average of its observed own signal $x_j$ and observed signal from the information provider $s_j$. The weight depends on firm $j$'s type $t_j$, precision of signal received from the information provider $\tau(t_j)$ and some parameters of the mechanism. When firm $i$ is taking expectation over firm $j$'s equilibrium strategy $E(q_j|x_i)$, it's equivalent for firm $i$ to calculate a weighted average of the expected own signal $E(x_j|x_i)$ and the expected signal from the information provider $E(s_j|x_i)$. Since signals are unbiased estimate of the true state, we have that $E(x_j|x_i) = E(s_j|x_i) = x_i$. Thus no matter what type firm $j$ is, what precision of signal firm $j$ receives from the information provider or what values parameters in the mechanism take, firm $i$'s best response function is the same as that of a game in which no firms participate to the mechansim or the information provider does not exist.

With Lemma 1 in hand, we are able to compute firms' interim profit before any signals are observed. Then the expected value from the information provider to a firm can be derived.

**Proposition 1.** *The expected value from the information provider to a firm with type $t$ when he is* *reporting $t'$ is given by*

$$V(t,t') = \begin{cases} \frac{1}{(2-d)^2}\frac{[1-r(t,t')A(d,\rho)]^2}{t[1-r(t,t')]} & for \quad t' \in \{k|a(k)=1\} \\ 0 & for \quad t' \in \{k|a(k)=0\} \end{cases}$$

*where* $A(d,\rho) = \frac{2-d\rho\int_{\{t|a(t)=1\}} dF(t)}{2-d\rho\int_{\{t|a(t)=1\}} r(t)dF(t)}$, $r(t,t') = \frac{t}{t+\tau(t')}$ *and* $r(t) = r(t,t) = \frac{t}{t+\tau(t)}$.

Notice that if $d \geq 0$, we have $\frac{\partial V}{\partial \tau} > 0$ for all types of firms. But if $d < 0$, $\frac{\partial V}{\partial \tau}$ can be positive or negative or zero for some type. This is due to the reason that there are two forces affecting firms' willingness to pay. One force is that receiving more accurate information aligns firms' actions with the true state. In any strategic environment, this force will lead firms to have higher willingness to pay for additional information with higher precision. The other force is that receiving more accurate information also aligns a firm's action with other firms' actions. This force has a positive effect when firms face a game of strategic complementarities and has a negative effect when firms face a game of strategic substitutes. Thus in an environment of strategic complementarities, firms have even greater willingness to pay for more accurate additional information. But when firms face a game of strategic substitutes, the willingness to pay is dampened since firms want their actions to be divergent. What's more, when the second force dominates the first one, some types of firms even have greater willingness to pay for addtional signals with lower precision.

In addition, the value of additional information is derived under the assumption that the mechanism satisfies (IC), (IR) and (NP). Thus we need to verify that the function derived indeed satisfies those constraints. We first look at the (IC) constraints. The following lemma verifies that the single-crossing property is satisfied.

**Lemma 2.** $\forall t \in \{k|a(k)=1\}$ *and* $\forall \tau$, *we have* $\frac{\partial^2 V(t,\tau)}{\partial t \partial \tau} < 0$.

We can define the expected (net) gain of a firm with type $t$ when he is reporting $t'$ as

$$U(t,t') = V(t,t') - m(t')$$

Denote $U(t) = U(t,t)$. Then Lemma 2 implies that (IC) is satisfied if and only if

$$U(t) = U(\underline{t}) + \int_{\underline{t}}^{t} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} \quad \forall t \in [\underline{t}, \bar{t}]$$

and $\tau(t)$ is nonincreasing in $t$ $\forall t \in \{k|a(k)=1\}$. Incorporating (IR) and (NP) constraints, we are able to identify sufficient and necessary conditions for a feasible menu. These conditions turn out to depend on the strategic environment that firms face.

**Proposition 2.** *If $d \geq 0$, a menu $\{(a(t), m(t))_{t\in[\underline{t},\bar{t}]}, (\tau(t))_{t\in\{k|a(k)=1\}}, \rho\}$ is feasible if and only if the following conditions are satisfied:*

*(a) $\exists t^* \in (\underline{t}, \bar{t}]$ such that $\{k|a(k)=1\} = [\underline{t}, t^*]$;*

*(b) $\tau(t)$ is nonincreasing in $t$ $\forall t \in [\underline{t}, t^*]$;*

*(c) $m(t) = 0$ $\forall t \in (t^*, \bar{t}]$ and $m(t) = V(t) + \int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}$ $\forall t \in [\underline{t}, t^*]$.*

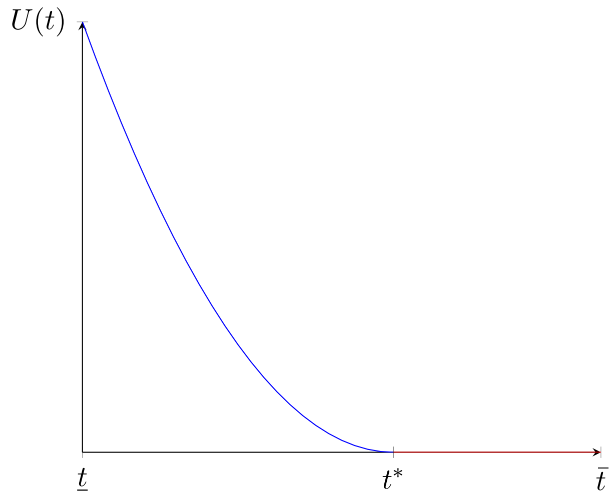



Figure 1: The expected net gain for different types when $d \geq 0$

Proposition 2 says that when firms face a game of strategic complementarities ($d > 0$) or no strategic interactions ($d = 0$), if the information provider chooses to exclude a subset of types, this subset must be an interval that starts from a cutoff type ($t^*$) that is indifferent between buying an additional signal or not to the highest precision type ($\bar{t}$). However, if firms face a game of strategic substitutes ($d < 0$), a feasible menu may have different features. This is described in the next proposition.

**Proposition 3.** *If $d < 0$, a menu $\{(a(t), m(t))_{t\in[\underline{t},\bar{t}]}, (\tau(t))_{t\in\{k|a(k)=1\}}, \rho\}$ is feasible if and only if either conditions ($a_1$) ($a_2$) ($a_3$) ($a_4$) are satisfied or conditions ($b_1$) ($b_2$) ($b_3$) ($b_4$) are satisfied:*

*($a_1$) $\exists t^* \in (\underline{t}, \bar{t}]$ such that $\{k|a(k)=1\} = [\underline{t}, t^*]$;*

*($a_2$) $\tau(t)$ is nonincreasing in $t$ $\forall t \in [\underline{t}, t^*]$;*

*($a_3$) $m(t) = 0$ $\forall t \in (t^*, \bar{t}]$ and $m(t) = V(t) + \int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}$ $\forall t \in [\underline{t}, t^*]$.*

*($a_4$) $r(t)A(d,\rho) \leq 1$ $\forall t \in [\underline{t}, t^*]$.*

*(b₁)* $\exists t^*, t^{**} \in (\underline{t}, \bar{t}]$ *such that* $t^* \leq t^{**}$ *and* $\{k|a(k) = 1\} = [\underline{t}, t^*] \cup [t^{**}, \bar{t}]$*;*

*(b₂)* $\tau(t)$ *is nonincreasing in* $t$ $\forall t \in [\underline{t}, t^*] \cup [t^{**}, \bar{t}]$*;*

*(b₃)* $m(t) = 0$ $\forall t \in (t^*, t^{**})$*,* $m(t) = V(t) + \int_t^{t^*} \frac{\partial V(\hat{t}, t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}$ $\forall t \in [\underline{t}, t^*]$ *and* $m(t) = V(t) - \int_{t^{**}}^{t} \frac{\partial V(\hat{t}, t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}$ $\forall t \in [t^{**}, \bar{t}]$.

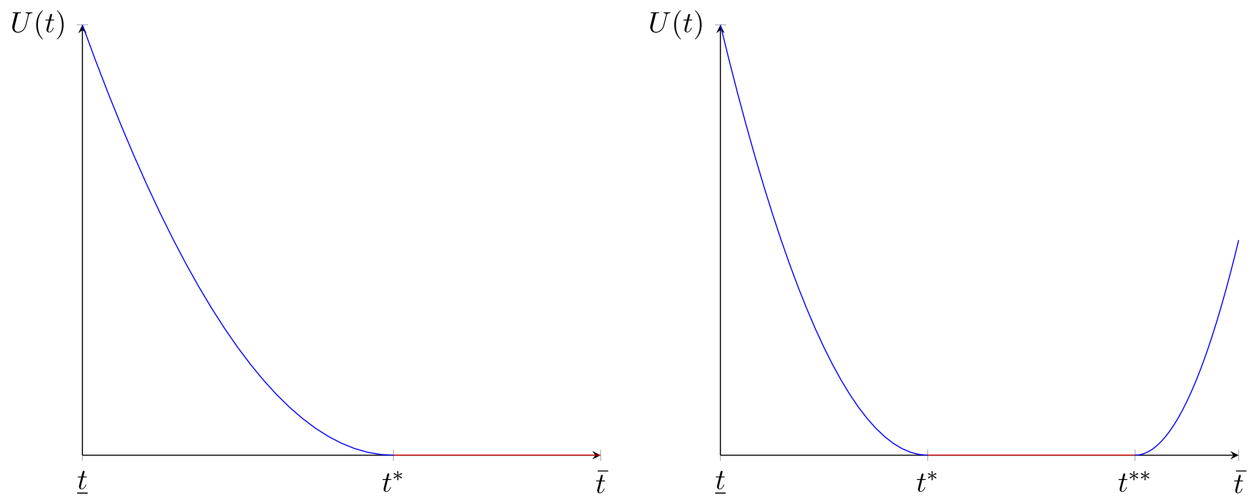
*(b₄)* $r(t)A(d, \rho) < 1$ $\forall t \in [\underline{t}, t^*]$ *and* $r(t)A(d, \rho) \geq 1$ $\forall t \in [t^{**}, \bar{t}]$.

Comparing Proposition 3 with Proposition 2 we can find that when $d < 0$, if the information provider imposes a menu such that $r(t)A(d, \rho) \leq 1$ $\forall t \in \{k|a(k) = 1\}$, like the case when $d \geq 0$, firms that may be excluded from the markets are those having relatively high precision of own signals. However, if $r(t)A(d, \rho) \geq 1$ for some types[5], these types have relatively high precision of own signals but are not excluded from the market. Instead, types that may be excluded are intermediate types. The intuition for this observation is that, for all kinds of strategic environments, firms with low types (having low precision of own signals) are prioritized by the information provider to participate in the market since they have relatively high willingness to pay for additional signals to acquire more precise information about the market demand. However, for the environment with strategic substitutes, some high types that put extremely high weight on their own signals ($r(t)A(d, \rho) \geq 1$) will not be excluded either. Compared to intermediate types, they are more willing to buy additional signals. This is because when firms face strategic substitutes, they want their actions more dispersed with each other. Obtaining an additional signal that is possibily correlated with the one received by other firms does less harm to firms that put extremely low weight on this additional signal compared to other types.

## 4 Optimal screening contract

In this section, we look for the optimal screening contract for the information provider under different strategic environments. We further want to see how the nature of competition between firms affects the information provider's optimal menu. The exact characterization of the optimal menu depends on the distribution of firms' types $F(.)$. To capture more insights, we assume that firms' types are uniformly distributed on $[\underline{t}, \bar{t}]$ and $\underline{t} \geq \frac{\bar{t}}{2}$. The assumption of uniform distribution is for tractability of the optimal menu. The assumption of $\underline{t} \geq \frac{\bar{t}}{2}$ is to guarantee that when firms face no strategic interaction ($d = 0$), all information structures provided are not distorted and the market is fully covered. Then we can isolate the effect of the nature of competition on the degree of informational distortion and the choice of optimal

[5] Since $\tau(t)$ is nondecreasing in $t$, $r(t)$ is increasing in $t$. Thus if $r(t)A(d, \rho) \geq 1$ for some $t$, it must true that $r(\hat{t})A(d, \rho) \geq 1$ $\forall \hat{t} > t$.




$(a_4)$ holds $(b_4)$ holds

Figure 2: The expected net gain for different types when $d < 0$

menu by the information provider. The following lemma shows that the optimal menu when firms face no strategic interactions is a single-item one.

**Lemma 3.** *Suppose $d = 0$ and firms' types are uniformly distributed on $[\underline{t}, \bar{t}]$ where $\frac{\bar{t}}{2} \leq \underline{t} < \bar{t}$. A menu $\{(a^*(t), m^*(t))_{t\in[\underline{t},\bar{t}]}, (\tau^*(t))_{t\in[t^*,\bar{t}]}, \rho^*\}$ maximizes the information provider's expected profit if $t^* = \bar{t}$ and $\tau^*(t) = \bar{\tau}\ \forall t \in [\underline{t}, \bar{t}]$.*

### 4.1 The optimal menu when $d > 0$

When $d > 0$, by Proposition 2, the information provider's problem is equivalent to the following program:

$$\max_{\{(\tau(t))_{t\in[\underline{t},t^*]},\rho,t^*\}} n \int_{\underline{t}}^{t^*} [V(t) + \int_{t}^{t^*} \frac{\partial V(\hat{t}, t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}] dF(t)$$

$$s.t. \qquad \tau(t) \text{ is nonincreasing in } t\ \forall t \in [\underline{t}, t^*]$$

In this program, the information provider will choose a cutoff type $t^*$ (possibly equal to $\bar{t}$) such that $a(t) = 0$, $m(t) = 0\ \forall t \in [t^*, \bar{t}]$ and $a(t) = 1$, $m(t) = V(t) + \int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}\ \forall t \in [\underline{t}, t^*]$. At the same time, $\forall t \in [\underline{t}, t^*]$, the information provider chooses $\rho$ and $\tau(t)$ to maximize the expected total payments from firms subject to the monotonicity constraint. The optimal menu is given in the following proposition.

**Proposition 4.** *Suppose $d > 0$ and firms' types are uniformly distributed on $[\underline{t}, \bar{t}]$ where $\frac{\bar{t}}{2} \leq \underline{t} < \bar{t}$. A menu $\{(a^*(t), m^*(t))_{t\in[\underline{t},\bar{t}]}, (\tau^*(t))_{t\in[\underline{t},t^*]}, \rho^*\}$ maximizes the information provider's expected profit if all the following conditions hold:*

*(a)* $t^* = \bar{t}$*;*

*(b)* $\rho^* = 1$*;*

*(c)* $\tau^*(t) = \bar{\tau}\ \forall t \in [\underline{t}, \bar{t}]$*.*

Notice that, just like the optimal menu under the environment of no strategic interactions, the information provider will offer a single-item menu with the most accurate information available. At the same time, the information provider will perfectly correlate the signals conditional on the true state. In this sense, the information provider is essentially sending public signals.

## 4.2 The optimal menu when $d < 0$

When $d < 0$, by Proposition 3, the information provider face two types of program. The first program is

$$\max_{\{(\tau(t))_{t\in[\underline{t},t^*]},\rho,t^*\}} n\int_{\underline{t}}^{t^*}[V(t)+\int_{t}^{t^*}\frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}}d\hat{t}]dF(t)$$

$$\begin{aligned} s.t. \qquad & \tau(t) \text{ is nonincreasing in } t\ \forall t \in [\underline{t}, t^*] \\ & r(t)A(d,\rho) \leq 1\ \forall t \in [\underline{t}, t^*] \end{aligned}$$

This program is the same with the one when $d > 0$ except for requiring an additional constraint. The solution for this program is given by the following proposition.

**Proposition 5.** *Suppose* $d < 0$ *and firms' types are uniformly distributed on* $[\underline{t}, \bar{t}]$ *where* $\frac{\bar{t}}{2} \leq \underline{t} < \bar{t}$*. If the information provider imposes a menu* $\{(a^*(t), m^*(t))_{t\in[\underline{t},\bar{t}]}, (\tau^*(t))_{t\in[\underline{t},t^*]}, \rho^*\}$ *such that* $r^*(t)A(d,\rho^*) \leq 1\ \forall t \in [\underline{t}, t^*]$*, then this menu maximizes the information provider's expected profit if all the following conditions hold:*

*(a)* $t^* = \bar{t}$*;*

*(b)* $\rho^* = 0$*;*

*(c)* $\tau^*(t) = \bar{\tau}\ \forall t \in [\underline{t}, \bar{t}]$*.*

For this program, the optimal menu will be such that all signals are provided independently with the highest precision. The second program is

$$\max_{\{(\tau(t))_{t\in[\underline{t},t^*]\cup[t^{**},\bar{t}]},\rho,t^*,t^{**}\}} n\int_{\underline{t}}^{t^*}[V(t)+\int_t^{t^*}\frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}}d\hat{t}]dF(t)+n\int_{t^{**}}^{\bar{t}}[V(t)-\int_{t^{**}}^{t}\frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}}d\hat{t}]dF(t)$$

$$\begin{aligned} s.t. \quad & \tau(t) \text{ is nonincreasing in } t\ \forall t\in[\underline{t},t^*]\cup[t^{**},\bar{t}] \\ & r(t)A(d,\rho)\leq 1\ \forall t\in[\underline{t},t^*] \\ & r(t)A(d,\rho)\geq 1\ \forall t\in[t^{**},\bar{t}] \\ & t^*\leq t^{**} \end{aligned}$$

The exact characterization for this program is hard but some properties of the optimal menu can be identified, which is given in the following proposition.

**Proposition 6.** *Suppose $d<0$ and firms' types are uniformly distributed on $[\underline{t},\bar{t}]$ where $\frac{\bar{t}}{2}\leq\underline{t}<\bar{t}$. If the information provider imposes a menu $\{(a^*(t),m^*(t))_{t\in[\underline{t},\bar{t}]},(\tau^*(t))_{t\in[\underline{t},t^*]\cup[t^{**},\bar{t}]},\rho^*\}$ such that $r^*(t)A(d,\rho^*)\leq 1\ \forall t\in[\underline{t},t^*)$ and $r^*(t)A(d,\rho^*)\geq 1\ \forall t\in[t^{**},\bar{t}]$, then this menu maximizes the information provider's expected profit* ***only if****:*

*(a) $t^*=t^{**}$;*

*(b) $r^*(t^*)A(d,\rho^*)=1$;*

*(c) $\tau^*(t)=\bar{\tau}\ \forall t\in[\underline{t},t^*)$ and $\tau^*(t)<\bar{\tau}\ \forall t\in[t^*,\bar{t}]$.*

For the second program, Proposition 6 (c) shows that optimally the information provider will partition the market into two segments: 'low type' segment and 'high type' segment. The information provider will offer the most accurate information to all types in the 'low type' segment but degrade and distort the information offered to types in the 'high type' segment. Proposition 6 (b) indicates that the information provider faces the trade-off between choosing how information structures are correlated ($\rho$), how the market is partitioned ($t^*$) and how information is distorted ($\tau(t)\ \forall t\in[t^*,\bar{t}]$). Since $\tau(t)$ is nonincreasing in $t$ and $A(d,\rho)$ is increasing in $\rho$ when $d<0$, if the information provider sets a relative high level for $\rho$ (which leads to a relative high level of $A(d,\rho)$), $r^*(t^*)$ should be set at a relatively low level. This leads to either a low level of $t^*$, which means the fraction of 'low type' segment is low, or a relatively low level of $\tau(t^*)$, which means the degree of informational distortion to 'high type' segment is high. A graphical illustration for the distortion is in Figure 3.

Among these two programs, the optimal menu in this case will be the one generating more expected profit for the information provider. Although the former one provides undistorted information to all

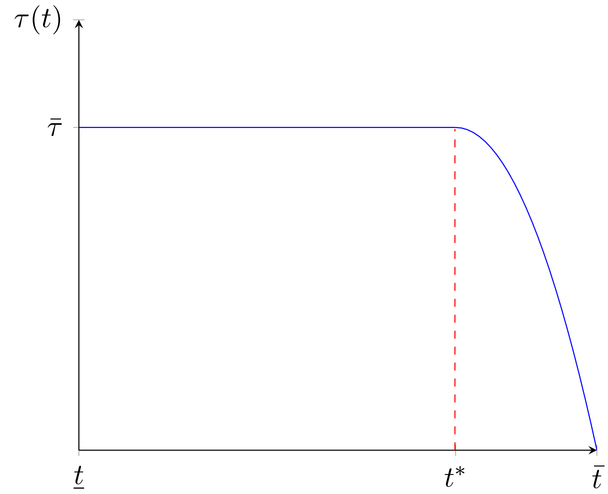



Figure 3: An optimal menu that distorts information $\forall t \in [t^*, \bar{t}]$ when $d < 0$

types in the market, it's not necessarily dominating the latter one. Since by restricting the fraction of 'low type' segment and providing degraded information to 'high type' segment, the information provider enhances the willingness to pay of very low types, whose surplus can be more heavily extracted. This intuition can be illustrated in Figure 4.

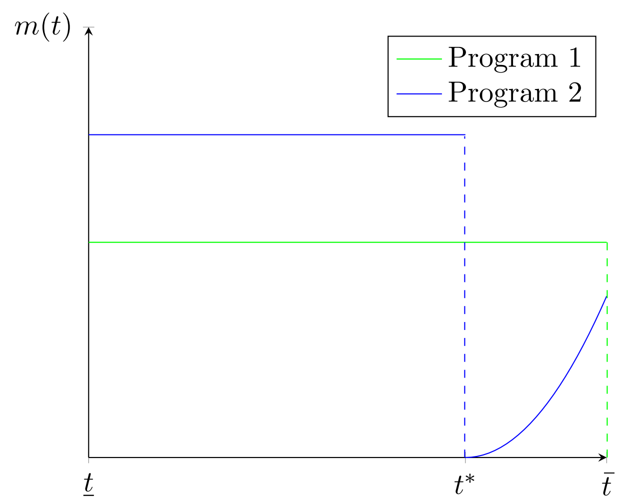



Figure 4: Comparison of payments for each type between the two programs when $d < 0$

# 5 Conclusions

In this paper, I study the design of feasible and optimal screening contracts for a monopolistic information provider in oligopoly. I study a sequential game in which the information provider first offers a menu of information structures, possibly correlated with a constant correlation coefficient. Then downstream firms compete conditional on each firm choosing an option from the menu (according to their types and willingness to pay) and forming posterior beliefs about the true state and other firms' actions. The mechanism design approach is applied to study the design of feasible menus that satisfy individual rationality constraints, incentive compatibility constraints and nonnegative payment constraints. Properties of a feasible menu under different strategic environments are identified. A feasible menu will always provide as accurate or more accurate information to low types of firms rather to high types of firms. When firms face strategic complementarities, their expected net gain of the additional information is nonincreasing in types and, thus, high types may be excluded from the market. When firms face strategic substitutes, a feasible menu may exclude intermediate types.

I also characterize the optimal menu when firms' types are uniformly distributed. I further study how the nature of competition between firms affects the information provider's optimal menu. Compared to an environment with no strategic interactions, firms facing strategic complementarities leads the information provider to perfectly correlate the information structures and provide most accurate information available. Instead, firms facing strategic substitutes may lead the market to be segmented and the information provided to high types degraded. In this case there is loss of efficiency in terms of information qualities.

Several interesting questions are left and may be conducted in the future research. First, what will be the optimal menu for a general distribution of types? Second, will it affect the design of menu if firms are facing nonlinear demand functions? Since the specification of linear demand functions guarantees a unique equilibrium and no learning from rejecting the menu, if demand functions are nonlinear, multiple equilibria may arise and the individual rationalities constraints may be modified to take into account the belief update when some type conjectures rejecting the menu.

## Appendix

**Proof of Lemma 1.**

We denote $B := \{k|a(k) = 1\}$. If $t_i \in B$, then conditional on observing a private signal $x_i$ and a signal from information provider $s_i \in S(t_i)$, firm $i$'s problem is to

$$\max_{q_i} E_{\theta,\boldsymbol{t}_{-i}}[(\theta + dq_{-i} - q_i)q_i|t_i, x_i, s_i, M]$$

The first order conditon gives firm $i$'s best response function as

$$q_i = \frac{1}{2}[E_{\theta,\boldsymbol{t}_{-i}}(\theta|t_i, x_i, s_i, M) + dE_{\theta,\boldsymbol{t}_{-i}}(q_{-i}|t_i, x_i, s_i, M)]$$

Under the normality assumption, we have that

$$E_\theta(\theta|t_i, x_i, s_i, M) = E_\theta(x_j|t_i, x_i, s_i, M) = \frac{t_i}{\tau(t_i)+t_i}x_i + \frac{\tau(t_i)}{\tau(t_i)+t_i}s_i \quad \forall j$$
$$E_\theta(s_j|t_i, x_i, s_i, M) = (1-\rho)\frac{t_i}{\tau(t_i)+t_i}x_i + [1-(1-\rho)\frac{t_i}{\tau(t_i)+t_i}]s_i \quad \forall j \in \{k|t_k \in B\} \ and \ j \neq i$$

If $t_i \notin B$, then conditional on observing a private signal $x_i$, firm $i$'s problem is to

$$\max_{q_i} E_{\theta,\boldsymbol{t}_{-i}}[(\theta + dq_{-i} - q_i)q_i|t_i, x_i, M]$$

The first order conditon is

$$q_i = \frac{1}{2}[E_{\theta,\boldsymbol{t}_{-i}}(\theta|t_i, x_i, M) + dE_{\theta,\boldsymbol{t}_{-i}}(q_{-i}|t_i, x_i, M)]$$

We also have

$$E_\theta(\theta|t_i, x_i, M) = E_\theta(x_j|t_i, x_i, M) = x_i \quad \forall j \neq i$$
$$E_\theta(s_j|t_i, x_i, M) = x_i \quad \forall j \in \{k|t_k \in B\} \ and \ j \neq i$$

Then we know that if $t_i \in B$ firm $i$'s best response function $q_i(t_i, x_i, s_i)$ is affine in $x_i$ and $s_i$ and if $t_i \notin B$ firm $i$'s best response function $q_i(t_i, x_i)$ is affine in $x_i$. We conjecture that

$$q_i^*(t_i, x_i, s_i) = A^1(t_i)x_i + A^2(t_i)s_i \qquad for \ \ t_i \in B$$

and

$$q_i^*(t_i, x_i) = A^3(t_i)x_i \qquad for \ \ t_i \notin B$$

Combinng each firm's best response function, we have the following equations:

$$\frac{1}{2}[\frac{t_i}{\tau(t_i)+t_i} + d(\int_B A^1(t)dF(t) + \int_{[\underline{t},\bar{t}]/B} A^3(t)dF(t))\frac{t_i}{\tau(t_i)+t_i} + d\int_B A^2(t)dF(t)(1-\rho)\frac{t_i}{\tau(t_i)+t_i}] = A^1(t_i) \quad \forall t_i \in B \tag{1}$$

$$\frac{1}{2}[\frac{\tau(t_i)}{\tau(t_i)+t_i}+d(\int_B A^1(t)dF(t)+\int_{[\underline{t},\bar{t}]/B} A^3(t)dF(t))\frac{\tau(t_i)}{\tau(t_i)+t_i}+d\int_B A^2(t)dF(t)[1-(1-\rho)\frac{t_i}{\tau(t_i)+t_i}]] = A^2(t_i) \quad \forall t_i \in B \tag{2}$$

$$\frac{1}{2}[1+d(\int_B A^1(t)dF(t)+\int_{[\underline{t},\bar{t}]/B} A^3(t)dF(t))+d\int_B A^2(t)dF(t)] = A^3(t_i) \quad \forall t_i \notin B \tag{3}$$

Add (1) and (2) from both sides gives

$$\frac{1}{2}[1+d\int_B (A^1(t)+A^2(t))dF(t)+d\int_{[\underline{t},\bar{t}]/B} A^3(t)dF(t))] = A^1(t_i)+A^2(t_i) \quad \forall t_i \in B \tag{4}$$

Compare (3) and (4), we have that

$$A^3(t_j) = A^1(t_i)+A^2(t_i) \quad \forall t_i \in B \; and \; \forall t_j \notin B \tag{5}$$

Plug (5) into (4) and take expectation over $t_j$ on both sides, we have

$$\int_{[\underline{t},\bar{t}]/B} A^3(t)dF(t) = \frac{1-\int_B dF(t)}{2-d} \tag{6}$$

and

$$\int_B (A^1(t)+A^2(t))dF(t) = \frac{\int_B dF(t)}{2-d} \tag{7}$$

Plug (6) and (7) back into (4), we get

$$A^3(t_j) = A^1(t_i)+A^2(t_i) = \frac{1}{2-d} \quad \forall t_i \in B \; and \; \forall t_j \notin B \tag{8}$$

Also, for (1) take the expectation of $t_i$ over the set $B$ on both sides we have

$$\frac{1}{2}\int_B (\frac{t}{\tau(t)+t})dF(t)[1+d\int_B (A^1(t))dF(t)+d(1-\rho)\int_B (A^2(t))dF(t)+d\frac{1-\int_B dF(t)}{2-d}] = \int_B (A^1(t))dF(t) \tag{9}$$

Combine (7) and (9) we can get that

$$\int_B (A^1(t))dF(t) = \frac{1}{2-d}\frac{2-d\rho\int_B dF(t)}{2-d\rho\int_B \frac{t}{t+\tau(t)}dF(t)}\int_B \frac{t}{t+\tau(t)}dF(t) \tag{10}$$

$$\int_B (A^2(t))dF(t) = \frac{1}{2-d}[\int_B dF(t) - \frac{2-d\rho\int_B dF(t)}{2-d\rho\int_B \frac{t}{t+\tau(t)}dF(t)}\int_B \frac{t}{t+\tau(t)}dF(t)] \tag{11}$$

Plug (10) and (11) back to (1) and (2) we get that

$$A^1(t) = \frac{1}{2-d} \frac{2 - d\rho \int_B dF(t)}{2 - d\rho \int_B \frac{t}{t+\tau(t)} dF(t)} \frac{t}{t+\tau(t)} \tag{12}$$

$$A^2(t) = \frac{1}{2-d}[1 - \frac{2 - d\rho \int_B dF(t)}{2 - d\rho \int_B \frac{t}{t+\tau(t)} dF(t)} \frac{t}{t+\tau(t)}] \tag{13}$$

Thus if $t_i \in B$, we have

$$q_i^*(t_i, x_i, s_i) = \frac{1}{2-d}[k(t_i, d, \rho)x_i + [1 - k(t_i, d, \rho)]s_i]$$

where $k(t_i, d, \rho) = \frac{2-d\rho \int_B dF(t)}{2-d\rho \int_B \frac{t}{t+\tau(t)} dF(t)} \frac{t_i}{t_i+\tau(t_i)}$.

If $t_i \notin B$, we have

$$q_i^*(t_i, x_i) = \frac{1}{2-d} x_i$$

□

**Proof of Proposition 1.**

Suppose firm $i$ with type $t_i$ deviates to report $t_i'$. If $a(t_i') = 0$, firm $i$'s problem is the same with a firm with type $t_i$ but not participating to the mechansim. Then we have

$$V(t, t') = \Pi^*(t, t'|a(t') = 0) - \hat{\Pi}(t) = \hat{\Pi}(t) - \hat{\Pi}(t) = 0 \quad if \quad a(t') = 0$$

If $a(t_i') = 1$, then conditional on observing a private signal $x_i$ and a signal from information provider $s_i \in S(t_i')$, firm $i$'s problem is to

$$\max_{q_i} E_{\theta, \boldsymbol{t}_{-i}}[(\theta + dq_{-i} - q_i)q_i | t_i, t_i', x_i, s_i, M]$$

The first order conditon gives firm $i$'s best response function as

$$q_i = \frac{1}{2}[E_{\theta, \boldsymbol{t}_{-i}}(\theta | t_i, t_i', x_i, s_i, M) + dE_{\theta, \boldsymbol{t}_{-i}}(q_{-i} | t_i, t_i', x_i, s_i, M)]$$

First, we have that

$$E_\theta(\theta|t_i,t_i',x_i,s_i,M)=E_\theta(x_j|t_i,t_i',x_i,s_i,M)=\frac{t_i}{\tau(t_i')+t_i}x_i+\frac{\tau(t_i')}{\tau(t_i)+t_i}s_i \quad \forall j$$
$$E_\theta(s_j|t_i,t_i',x_i,s_i,M)=(1-\rho)\frac{t_i}{\tau(t_i')+t_i}x_i+[1-(1-\rho)\frac{t_i}{\tau(t_i')+t_i}]s_i \quad \forall j\in\{k|a(t_k)=1\}\ and\ j\neq i$$

Also, by Lemma 1 we know that

$$q_j^*(t_j,x_j,s_j)=\frac{1}{2-d}[k(t_j,d,\rho)x_j+[1-k(t_j,d,\rho)]s_j] \quad \forall j\in\{k|a(t_k)=1\}\ and\ j\neq i$$

$$q_j^*(t_j,x_j)=\frac{1}{2-d}x_j \quad \forall j\in\{k|a(t_k)=0\}\ and\ j\neq i$$

where $k(t_j,d,\rho)=\frac{2-d\rho\int_{\{t|a(t)=1\}}dF(t)}{2-d\rho\int_{\{t|a(t)=1\}}\frac{t}{t+\tau(t)}dF(t)}\frac{t_j}{t_j+\tau(t_j)}$.

Thus firm $i$'s strategy is given by

$$\begin{aligned}q_i^*(x_i,s_i,t_i,t_i')=&\frac{1}{2}[\frac{t_i}{\tau(t_i')+t_i}x_i+\frac{\tau(t_i')}{\tau(t_i)+t_i}s_i+d\frac{1}{2-d}[\int_{\{t|a(t)=1\}}(k(t_j,d,\rho))dF(t_j)x_i\frac{t_i}{\tau(t_i')+t_i}+\\&(\int_{\{t|a(t)=1\}}dF(t)-\int_{\{t|a(t)=1\}}(k(t_j,d,\rho))dF(t_j))[(1-\rho)\frac{t_i}{\tau(t_i')+t_i}x_i+[1-(1-\rho)\frac{t_i}{\tau(t_i')+t_i}]s_i]\\&+\frac{t_i}{\tau(t_i')+t_i}x_i\int_{\{t|a(t)=0\}}dF(t)+\frac{\tau(t_i')}{\tau(t_i)+t_i}s_i\int_{\{t|a(t)=0\}}dF(t)]]\end{aligned}$$

Simplify the expression and we get that

$$q_i^*(x_i,s_i,t_i,t_i')=\frac{1}{2-d}[l(t_i,t_i',d,\rho)x_i+(1-l(t_i,t_i',d,\rho))s_i]$$

where $l(t_i,t_i',d,\rho)=\frac{2-d\rho\int_B dF(t)}{2-d\rho\int_B\frac{t}{t+\tau(t)}dF(t)}\frac{t_i}{t_i+\tau(t_i')}$.

Thus, conditional on the true state $\theta$ the firm's expected profit before observing signals is given by

$$
\begin{aligned}
\Pi_i^*(t_i, t_i'|\theta) &= E_{x_i,s_i}[(\theta + dq_{-i}^* - q_i^*)q_i^*|\theta] \\
&= E_{x_i,s_i}[[\theta + \frac{d}{2-d}\int_{\{t|a(t)=1\}}(k(t_j,d,\rho)x_j + (1-k(t_j,d,\rho))s_j)dF(t_j) + \frac{d}{2-d}\int_{\{t|a(t)=0\}} x_j dF(t_j) \\
&\quad - \frac{1}{2-d}(l(t_i,t_i',d,\rho)x_i + (1-l(t_i,t_i',d,\rho))s_i)]\frac{1}{2-d}(l(t_i,t_i',d,\rho)x_i + (1-l(t_i,t_i',d,\rho))s_i)|\theta] \\
&= \frac{1}{(2-d)}[\theta^2 + \frac{d}{2-d}\theta^2 + \frac{d}{2-d}\frac{\int_{\{t|a(t)=1\}}(1-k(t_j,d,\rho))dF(t_j)(1-l(t_i,t_i',d,\rho))\rho}{\tau(t_i')} \\
&\quad - \frac{1}{2-d}\theta^2 - \frac{1}{2-d}\frac{l^2(t_i,t_i',d,\rho)}{t_i} - \frac{1}{2-d}\frac{(1-l(t_i,t_i',d,\rho))^2}{\tau(t_i')}] \\
&= \frac{1}{(2-d)^2}[\theta^2 - \frac{l^2(t_i,t_i',d,\rho)}{t_i} - \frac{(1-l(t_i,t_i',d,\rho))^2}{\tau(t_i')} + \frac{(1-l(t_i,t_i',d,\rho))}{\tau(t_i')}2(1-A(d,\rho))]
\end{aligned}
$$

The last equality comes from the fact that

$$
d\rho\int_{\{t|a(t)=1\}}(1-k(t_j,d,\rho))dF(t_j) = 2d\rho\frac{\int_{\{t|a(t)=1\}}dF(t) - \int_{\{t|a(t)=1\}}r(t)dF(t)}{2 - d\rho\int_{\{t|a(t)=1\}}r(t)dF(t)} = 2(1-A(d,\rho))
$$

Now if the firm does not participate to mechanism, its expected profit conditional on the true state $\theta$ before observing signals is

$$
\begin{aligned}
\hat{\Pi}_i(t_i|\theta) &= E_{x_i}[(\theta + dq_{-i}^* - q_i^*)q_i^*|\theta] \\
&= E_{x_i}[[\theta + \frac{d}{2-d}\int_{\{t|a(t)=1\}}(k(t_j,d,\rho)x_j + (1-k(t_j,d,\rho))s_j)dF(t_j) \\
&\quad + \frac{d}{2-d}\int_{\{t|a(t)=0\}}x_j dF(t_j) - \frac{1}{2-d}x_i]\frac{1}{2-d}x_i|\theta] \\
&= \frac{1}{2-d}[\theta^2 + \frac{d}{2-d}\theta^2 - \frac{1}{2-d}\theta^2 - \frac{1}{2-d}\frac{1}{t_i}] \\
&= \frac{1}{(2-d)^2}[\theta^2 - \frac{1}{t_i}]
\end{aligned}
$$

Now we can derive the expected value from the additional informaiton provided if a firm with type $t$ reports $t'$ (we ignore the subscript) as

$$
\begin{aligned}
V(t,t') &= E_\theta[\Pi^*(t,t'|\theta, a(t')=1) - \hat{\Pi}(t|\theta)] \\
&= \frac{1}{(2-d)^2}E_\theta[\theta^2 - \frac{l^2(t,t',d,\rho)}{t} - \frac{(1-l(t,t',d,\rho))^2}{\tau(t')} + \frac{(1-l(t,t',d,\rho))}{\tau(t')}2(1-A(d,\rho)) - \theta^2 + \frac{1}{t}] \\
&= \frac{1}{(2-d)^2}\frac{[1-r(t,t')A(d,\rho)]^2}{t[1-r(t,t')]}
\end{aligned}
$$

where $A(d,\rho) = \frac{2-d\rho\int_{\{t|a(t)=1\}} dF(t)}{2-d\rho\int_{\{t|a(t)=1\}} \frac{t}{t+\tau(t)}dF(t)}$ and $r(t,t') = \frac{t}{t+\tau(t')}$.

$\square$

**Proof of Lemma 2.**

$\forall t \in \{k|a(k)=1\}$ and $\forall \tau$, we have $\frac{\partial V(t,\tau)}{\partial t} = \frac{1}{(2-d)^2}\frac{t(1-r)2(1-rA)(-A)\frac{\partial r}{\partial t} - (1-rA)^2(1-t\frac{\partial r}{\partial t}-r)}{t^2(1-r)^2} = \frac{1}{(2-d)^2}\frac{r^2A^2-1}{t^2}$. Then we have $\frac{\partial^2 V(t,\tau)}{\partial t \partial \tau} = \frac{1}{(2-d)^2}\frac{2A^2 r}{t^2}\frac{\partial r}{\partial \tau} = -\frac{2}{(2-d)^2}\frac{A^2r^3}{t^3} < 0$.

$\square$

**Proof of Proposition 2.**

**Sufficiency:**

(i) (c) implies that $U(t) = 0\ \forall t \in (t^*, \bar{t}]$ and $U(t) = -\int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}\ \forall t \in [\underline{t}, t^*]$. Thus we have $U(t) = U(\underline{t}) + \int_{\underline{t}}^{t} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} \quad \forall t \in [\underline{t}, \bar{t}]$. Combined with (b), (IC) is satisfied.

(ii) $\forall t \in [\underline{t}, t^*]$, we have

$$\frac{\partial V(t,t')}{\partial t}|_{t'=t} = \frac{1}{(2-d)^2}\frac{r^2(t)A^2(d,\rho)-1}{t^2} < 0$$

Thus $V(t) - m(t) = -\int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} > 0\ \forall t \in [\underline{t}, t^*]$. Thus (IR) is satisfied.

(iii) $\forall t \in [\underline{t}, t^*]$, we have

$$\frac{\partial V(t,t')}{\partial t'}|_{t'=t} = \frac{1}{(2-d)^2}\frac{(r(t)A(d,\rho)-1)(r(t)A(d,\rho)-2A(d,\rho)-1)}{\tau^2(t)}\frac{\partial \tau(t)}{\partial t}$$

First we have $(r(t)A(d,\rho)-1)(r(t)A(d,\rho)-2A(d,\rho)-1) > 0$. Second, by (b) we have $\frac{\partial \tau(t)}{\partial t} \leq 0$. Thus $\frac{\partial V(t,t')}{\partial t'}|_{t'=t} \leq 0$. Then we have $m(t) = V(t) + \int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} = V(t^*) - \int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial t'}|_{t'=\hat{t}} d\hat{t} \geq V(t^*) \geq 0$ which implies that (NP) is satisfied.

**Necessity:**

(i) (IC) implies that $U(t) = U(\underline{t}) + \int_{\underline{t}}^{t} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}\ \forall t \in [\underline{t}, \bar{t}]$ and $\tau(t)$ is nonincreasing in $t$ $\forall t \in \{k|a(k)=1\}$. Since $\frac{\partial V(t,t')}{\partial t}|_{t'=t} \leq 0$ we have $U(t)$ is nonincreasing in $t$.

(ii) By (NP) and (IR) We know that if $t \in \{k|a(k)=0\}$, $m(t) \geq 0$ and $V(t) - m(t) = 0 - m(t) \geq 0$. Thus $m(t) = 0$ and $U(t) = V(t) - m(t) = 0 - 0 = 0$. By (IR), we have that $U(t) \geq 0\ \forall t \in [\underline{t}, \bar{t}]$. Also by (i) we have that $U(t)$ is absolutely continuous and nonincreasing in $t$. Thus if $\{k|a(k)=0\} \neq \varnothing$, there must exist $t^* \in (\underline{t}, \bar{t})$ such that $\{k|a(k)=0\} = [t^*, \bar{t}]$. Thus (a) is satisfied.

(iii) (b) and (c) are immediately satisfied by (IC) and (ii).

$\square$

**Proof of Proposition 3.**

We first prove that it can't be true that $r(t)A(d,\rho) \geq 1 \ \forall t \in \{k|a(k)=1\}$. Suppose this is true, we require that

$$r(t)A(d,\rho) = \frac{t}{t+\tau(t)} \frac{2 - d\rho \int_{\{t|a(t)=1\}} dF(t)}{2 - d\rho \int_{\{t|a(t)=1\}} \frac{t}{t+\tau(t)} dF(t)} \geq 1 \quad \forall t \in \{k|a(k)=1\}$$

Take expectation of $t$ over the set $\{k|a(k)=1\}$ on both sides, we get

$$\int_{\{t|a(t)=1\}} \frac{t}{t+\tau(t)} dF(t) \frac{2 - d\rho \int_{\{t|a(t)=1\}} dF(t)}{2 - d\rho \int_{\{t|a(t)=1\}} \frac{t}{t+\tau(t)} dF(t)} \geq \int_{\{t|a(t)=1\}} dF(t)$$

This is equivalent to

$$\int_{\{t|a(t)=1\}} \frac{t}{t+\tau(t)} dF(t) \geq \int_{\{t|a(t)=1\}} dF(t)$$

This holds only if $\tau(t) = 0 \quad \forall t \in \{k|a(k)=1\}$, thus contradiction. Thus either $r(t)A(d,\rho) < 1 \ \forall t \in \{k|a(k)=1\}$ or $r(t)A(d,\rho) < 1$ for some types in $\{k|a(k)=1\}$ and $r(t)A(d,\rho) \geq 1$ for other types in $\{k|a(k)=1\}$.

**Sufficiency for** $(a_1)$ $(a_2)$ $(a_3)$ $(a_4)$**:**

$(a_1)$ and $(a_4)$ implies that $r(t)A(d,\rho) < 1 \ \forall t \in \{k|a(k)=1\}$. Then the proof is exactly the same as the proof of sufficiency in Proposition 2.

**Necessity for** $(a_1)$ $(a_2)$ $(a_3)$ $(a_4)$**:**

Assume that $r(t)A(d,\rho) < 1 \ \forall t \in \{k|a(k)=1\}$. Then the proof is exactly the same as the proof of necessity in Proposition 2. If this condition does not hold, it will lead to the necessity of $(b_1)$ $(b_2)$ $(b_3)$ $(b_4)$ which will be proved later.

**Sufficiency for** $(b_1)$ $(b_2)$ $(b_3)$ $(b_4)$**:**

(i) $(b_3)$ implies that $U(t) = 0 \ \forall t \in (t^*, t^{**})$, $U(t) = -\int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} \ \forall t \in [\underline{t}, t^*]$ and $U(t) = \int_{t^{**}}^t \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} \ \forall t \in [t^{**}, \bar{t}]$. Thus we have $U(t) = U(\underline{t}) + \int_{\underline{t}}^t \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} \quad \forall t \in [\underline{t}, \bar{t}]$. Combined with $(b_2)$, (IC) is satisfied.

(ii) $\forall t \in [\underline{t}, t^*]$, we have $\frac{\partial V(t,t')}{\partial t}|_{t'=t} < 0$. Thus $V(t) - m(t) = -\int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} > 0 \ \forall t \in [\underline{t}, t^*]$. $\forall t \in [t^{**}, \bar{t}]$, we have $\frac{\partial V(t,t')}{\partial t}|_{t'=t} \geq 0$. Thus $V(t) - m(t) = \int_{t^{**}}^t \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} \geq 0 \ \forall t \in [t^{**}, \bar{t}]$. Thus (IR) is satisfied.

(iii) $\forall t \in [\underline{t}, t^*]$, we have $\frac{\partial V(t,t')}{\partial t'}|_{t'=t} \leq 0$. Then we have $m(t) = V(t) + \int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} = V(t^*) - \int_t^{t^*} \frac{\partial V(\hat{t},t')}{\partial t'}|_{t'=\hat{t}} d\hat{t} \geq V(t^*) \geq 0 \ \forall t \in [\underline{t}, t^*]$. $\forall t \in [t^{**}, \bar{t}]$, we have $\frac{\partial V(t,t')}{\partial t'}|_{t'=t} \geq 0$. Then we have $m(t) =$

$V(t) - \int_{t^{**}}^{t} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t} = V(t^{**}) + \int_{t^{**}}^{t} \frac{\partial V(\hat{t},t')}{\partial t'}|_{t'=\hat{t}} d\hat{t} \geq V(t^{**}) \geq 0\ \forall t \in [t^{**}, \bar{t}]$. Thus (NP) is satisfied.

**Necessity for $(b_1)$ $(b_2)$ $(b_3)$ $(b_4)$:**

Assume $r(t)A(d,\rho) < 1$ for some types in $\{k|a(k) = 1\}$ and $r(t)A(d,\rho) \geq 1$ for other types in $\{k|a(k) = 1\}$. If this does not hold, it will lead to the necessity of $(a_1)$ $(a_2)$ $(a_3)$ $(a_4)$.

(i) (IC) implies that $U(t) = U(\underline{t}) + \int_{\underline{t}}^{t} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}\ \forall t \in [\underline{t}, \bar{t}]$ and $\tau(t)$ is nonincreasing in $t$ $\forall t \in \{k|a(k) = 1\}$. Since $r'(t) = \frac{\tau(t) - t\tau'(t)}{(t+\tau(t))^2}$, we have $r(t)$ is increasing in $t$ $\forall t \in \{k|a(k) = 1\}$. Thus if $r(\hat{t})A(d,\rho) \geq 1$ for some $\hat{t} \in \{k|a(k) = 1\}$, it must be true that $r(t)A(d,\rho) \geq 1\ \forall t > \hat{t}$ and $t \in \{k|a(k) = 1\}$. Suppose $t^{**}$ is the lowest type in $\{k|a(k) = 1\}$ such that $r(t^{**})A(d,\rho) \geq 1$. Also, we have that if $r(\hat{t})A(d,\rho) < 1$ for some $\hat{t} \in \{k|a(k) = 1\}$, it must be true that $r(t)A(d,\rho) < 1\ \forall t < \hat{t}$ and $t \in \{k|a(k) = 1\}$. Suppose $t^*$ is the highest type in $\{k|a(k) = 1\}$ such that $r(t^*)A(d,\rho) < 1$.

(ii) $\forall t \leq t^*$ and $t \in \{k|a(k) = 1\}$, we have $\frac{\partial V(t,t')}{\partial t}|_{t'=t} \leq 0$. Thus we have $U(t)$ is nonincreasing in $t$ $\forall t \leq t^*$ and $t \in \{k|a(k) = 1\}$. $\forall t \geq t^{**}$ and $t \in \{k|a(k) = 1\}$, we have $\frac{\partial V(t,t')}{\partial t}|_{t'=t} \geq 0$. Thus we have $U(t)$ is nondecreasing in $t$ $\forall t \geq t^{**}$ and $t \in \{k|a(k) = 1\}$

(iii) By (NP) and (IR) We know that if $t \in \{k|a(k) = 0\}$, $m(t) \geq 0$ and $V(t) - m(t) = 0 - m(t) \geq 0$. Thus $m(t) = 0$ and $U(t) = V(t) - m(t) = 0 - 0 = 0$. By (IR), we have that $U(t) \geq 0\ \forall t \in [\underline{t}, \bar{t}]$. Also by (i) we know that if $\{k|a(k) = 0\} \neq \varnothing$, it must be true that $\{k|a(k) = 0\} = [t^*, t^{**}]$. Thus $(a_1)$ is satisfied.

(iv) $(a_2)$ $(a_3)$ $(a_4)$ are immediately satisfied by (IC) and (iii).

□

**Proof of Lemma 3.**

The information provider's expected profit can be represented by

$$
\begin{aligned}
\Pi(t^*, \tau(.)) &= n \int_{\underline{t}}^{t^*} [V(t) + \int_{t}^{t^*} \frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}} d\hat{t}] dF(t) \\
&= n \int_{\underline{t}}^{t^*} [V(t) + \frac{\partial V(t,t')}{\partial t}|_{t'=t} \frac{F(t)}{f(t)}] dF(t) \\
&= n \int_{\underline{t}}^{t^*} [V(t) + \frac{\partial V(t,t')}{\partial t}|_{t'=t} (t - \underline{t})] dF(t) \\
&= \frac{n}{(2-d)^2} \int_{\underline{t}}^{t^*} [\frac{1-r(t)}{t} + \frac{r^2(t)-1}{t^2}(t - \underline{t})] dF(t)
\end{aligned}
$$

(i) Denote $H(t,\tau(t)) = \frac{1-r(t)}{t} + \frac{r^2(t)-1}{t^2}(\bar{t} - t)$. Maximizing $H(t,\tau(t))$ for each $t \in [\underline{t}, t^*]$ is a pointwise maximization of $\Pi(t^*, \tau(.))$. For this, we have the first order derivative of $H(t,\tau(t))$ with respect to $\tau(t)$

as

$$\frac{\partial H(t,\tau(t))}{\partial \tau(t)} = \frac{r^2(t)}{t^2} - 2\frac{r^3(t)}{t^3}(t-\underline{t}) \geq \frac{r^2(t)}{t^2} - 2\frac{r^3(t)}{t^3}(t-\frac{t}{2}) = \frac{r^2(t)}{t^2}(1-r(t)) > 0$$

Thus $\tau^*(t) = \bar{\tau}$ and this automatically satisfies monotonicity.

(ii) The first order derivative of $\Pi(t^*, \rho, \tau(.))$ with respect to $t^*$ is given by

$$\begin{aligned}
\frac{\partial \Pi(t^*,\rho,\tau(.))}{\partial t^*} &= \frac{n}{(2-d)^2}[\frac{1-r(t^*)}{t^*} + \frac{r^2(t^*)-1}{t^{*2}}(t^*-\underline{t})] \\
&= \frac{n(1-r(t^*))}{(2-d)^2 t^{*2}}(\underline{t} + r(t^*)\underline{t} - r(t^*)t^*) \\
&\geq \frac{n(1-r(t^*))}{(2-d)^2 t^{*2}}(\underline{t} + r(t^*)r\underline{t} - 2r(t^*)\underline{t}) \\
&= \frac{n(1-r(t^*))^2}{(2-d)^2 t^{*2}}\underline{t} \\
&> 0
\end{aligned}$$

Thus the information provider will set $t^* = \bar{t}$.

$\square$

**Proof of Proposition 4.**

The information provider's expected profit can be represented by

$$\begin{aligned}
\Pi(t^*,\rho,\tau(.)) &= n\int_{\underline{t}}^{t^*}[V(t) + \int_t^{t^*}\frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}}d\hat{t}]dF(t) \\
&= n\int_{\underline{t}}^{t^*}[V(t) + \frac{\partial V(t,t')}{\partial t}|_{t'=t}\frac{F(t)}{f(t)}]dF(t) \\
&= n\int_{\underline{t}}^{t^*}[V(t) + \frac{\partial V(t,t')}{\partial t}|_{t'=t}(t-\underline{t})]dF(t) \\
&= \frac{n}{(2-d)^2}\int_{\underline{t}}^{t^*}[\frac{[1-r(t)A(d,\rho)]^2}{t[1-r(t)]} + \frac{r^2(t)A^2(d,\rho)-1}{t^2}(t-\underline{t})]dF(t)
\end{aligned}$$

(i) The first order derivative with respect to $\rho$ is given by

$$\frac{\partial \Pi(t^*,\rho,\tau(.))}{\partial \rho} = \frac{n}{(2-d)^2}\frac{\partial A(d,\rho)}{\partial \rho}\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]} + \frac{r^2(t)2A(d,\rho)}{t^2}(t-\underline{t})]dF(t)$$

We have $\frac{\partial A(d,\rho)}{\partial \rho} = \frac{2d\int_{\underline{t}}^{t^*}(r(t)-1))dF(t)}{(2-d\rho\int_{\underline{t}}^{t^*}r(t)dF(t))^2} < 0$. Also, $\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]} + \frac{r^2(t)2A(d,\rho)}{t^2}(t-\underline{t})]dF(t) < \int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]} + \frac{r^2(t)2A(d,\rho)}{t^2}(t-\frac{t}{2})]dF(t) = \int_{\underline{t}}^{t^*}\frac{r(t)[A(d,\rho)r(t)(3-r(t))-2]}{t[1-r(t)]}dF(t) < \int_{\underline{t}}^{t^*}\frac{r(t)[2A(d,\rho)-2]}{t[1-r(t)]}dF(t) < 0$. Thus $\frac{\partial \Pi(t^*,\rho,\tau(.))}{\partial \rho} > 0$ and the information provider will set $\rho^* = 1$.

(ii) Denote $H(t,\tau(t)) = V(t) + \frac{\partial V(t,t')}{\partial t}|_{t'=t}\frac{F(t)}{f(t)} = V(t,\tau(t)) + \frac{\partial V(t,\tau(t'))}{\partial t}|_{t'=t}\frac{F(t)}{f(t)}$. Maximizing $H(t,\tau(t))$

for each for each $t \in [\underline{t}, t^*]$ is a pointwise maximization of $\Pi(t^*, \rho, \tau(.))$. For this, we have the first order derivative of $H(t, \tau(t))$ with respect to $\tau(t)$ as

$$\begin{aligned}
\frac{\partial H(t,\tau(t))}{\partial \tau(t)} &= \frac{1}{(2-d)^2}[\frac{(r(t)A(d,\rho)-1)(r(t)A(d,\rho)-2A(d,\rho)-1)}{\tau^2(t)} - \frac{2r^3(t)A^2(d,\rho)}{t^3}(t-\underline{t})] \\
&\geq \frac{1}{(2-d)^2}[\frac{(r(t)A(d,\rho)-1)(r(t)A(d,\rho)-2A(d,\rho)-1)}{\tau^2(t)} - \frac{2r^3(t)A^2(d,\rho)}{t^3}\frac{t}{2}] \\
&= \frac{1}{(2-d)^2}\frac{r(t)^2}{t^2(1-r(t))^2}(3r^2A^2 - r^3A^2 - 3rA^2 - 2rA + 2A + 1) \\
&> \frac{1}{(2-d)^2}\frac{r(t)^2}{t^2(1-r(t))^2}(rA^2(3r - r^2 - 3) - 2rA + 2A + 1) \\
&\geq \frac{1}{(2-d)^2}\frac{r(t)^2}{t^2(1-r(t))^2}(-A^2 - 2rA + 2A + 1) \\
&\geq \frac{1}{(2-d)^2}\frac{r(t)^2}{t^2(1-r(t))^2}(-A^2 + 1) \\
&> 0
\end{aligned}$$

We have $\frac{\partial H(t,\tau(t))}{\partial \tau(t)} > 0$. Thus $\tau^*(t) = \bar{\tau}$ and this automatically satisfies monotonicity.

(iii) The first order derivative of $\Pi(t^*, \rho, \tau(.))$ with respect to $t^*$ is given by

$$\frac{\partial \Pi(t^*,\rho,\tau(.))}{\partial t^*} = \frac{n}{(2-d)^2}[\frac{[1-r(t^*)A(d,\rho)]^2}{t^*[1-r(t^*)]} + \frac{r^2(t^*)A^2(d,\rho)-1}{t^{*2}}(t^* - \underline{t})]$$

By $\underline{t} > \frac{t^*}{2}$, easy to check that this is positive. Thus $\frac{\partial \Pi(t^*,\rho,\tau(.))}{\partial t^*} > 0$ and the information provider will set $t^* = \bar{t}$.

$\square$

**Proof of Proposition 5.**

The information provider's expected profit can be represented by

$$\begin{aligned}
\Pi(t^*, \rho, \tau(.)) &= n\int_{\underline{t}}^{t^*}[V(t) + \int_{t}^{t^*}\frac{\partial V(\hat{t},t')}{\partial \hat{t}}|_{t'=\hat{t}}d\hat{t}]dF(t) \\
&= n\int_{\underline{t}}^{t^*}[V(t) + \frac{\partial V(t,t')}{\partial t}|_{t'=t}\frac{F(t)}{f(t)}]dF(t) \\
&= n\int_{\underline{t}}^{t^*}[V(t) + \frac{\partial V(t,t')}{\partial t}|_{t'=t}(t-\underline{t})]dF(t) \\
&= \frac{n}{(2-d)^2}\int_{\underline{t}}^{t^*}[\frac{[1-r(t)A(d,\rho)]^2}{t[1-r(t)]} + \frac{r^2(t)A^2(d,\rho)-1}{t^2}(t-\underline{t})]dF(t)
\end{aligned}$$

The first order derivative with respect to $\rho$ is given by

$$\frac{\partial\Pi(t^*,\rho,\tau(.))}{\partial\rho}=\frac{n}{(2-d)^2}\frac{\partial A(d,\rho)}{\partial\rho}\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]}+\frac{r^2(t)2A(d,\rho)}{t^2}(t-\underline{t})]dF(t)$$

We have $\frac{\partial A(d,\rho)}{\partial\rho}=\frac{2d\int_{\underline{t}}^{t^*}(r(t)-1))dF(t)}{(2-d\rho\int_{\underline{t}}^{t^*}r(t)dF(t))^2}>0$. Thus $\frac{\partial\Pi(t^*,\rho,\tau(.))}{\partial\rho}|_{\rho=0}=\frac{n}{(2-d)^2}\frac{\partial A(d,\rho)}{\partial\rho}|_{\rho=0}\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)]}{t[1-r(t)]}+\frac{r^2(t)2}{t^2}(t-\underline{t})]dF(t)$. We have $\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)]}{t[1-r(t)]}+\frac{r^2(t)2}{t^2}(t-\underline{t})]dF(t)<\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)]}{t[1-r(t)]}+\frac{r^2(t)2}{t^2}(t-\frac{t}{2})]dF(t)=\int_{\underline{t}}^{t^*}\frac{r(t)[r(t)(3-r(t))-2]}{t[1-r(t)]}dF(t)<\int_{\underline{t}}^{t^*}\frac{r(t)[2-2]}{t[1-r(t)]}dF(t)=0$. Thus $\frac{\partial\Pi(t^*,\rho,\tau(.))}{\partial\rho}|_{\rho=0}<0$ and the information provider find its optimal to set $\rho=0$. Then $A(d,\rho)=1$ and the constraints $r(t)A(d,\rho)\le 1\ \forall t\in[\underline{t},t^*]$ are satisfied. Then the information provider's problem is the same as the case when $d=0$, which means $t^*=\bar{t}$ and $\tau^*(t)=\bar{\tau}\ \forall t\in[\underline{t},\bar{t}]$.

□

**Proof of Proposition 6.**

The information provider's expected profit can be represented by

$$\begin{aligned}\Pi(t^*,t^{**},\rho,\tau(.))&=n\int_{\underline{t}}^{t^*}[V(t)+\int_t^{t^*}\frac{\partial V(\hat{t},t')}{\partial\hat{t}}|_{t'=\hat{t}}d\hat{t}]dF(t)+n\int_{t^{**}}^{\bar{t}}[V(t)-\int_{t^{**}}^{t}\frac{\partial V(\hat{t},t')}{\partial\hat{t}}|_{t'=\hat{t}}d\hat{t}]dF(t)\\&=n\int_{\underline{t}}^{t^*}[V(t)+\frac{\partial V(t,t')}{\partial t}|_{t'=t}\frac{F(t)}{f(t)}]dF(t)+n\int_{t^{**}}^{\bar{t}}[V(t)-\frac{\partial V(t,t')}{\partial t}|_{t'=t}\frac{(1-F(t))}{f(t)}]dF(t)\\&=n\int_{\underline{t}}^{t^*}[V(t)+\frac{\partial V(t,t')}{\partial t}|_{t'=t}(t-\underline{t})]dF(t)+n\int_{t^{**}}^{\bar{t}}[V(t)-\frac{\partial V(t,t')}{\partial t}|_{t'=t}(\bar{t}-t)]dF(t)\\&=\frac{n}{(2-d)^2}\int_{\underline{t}}^{t^*}[\frac{[1-r(t)A(d,\rho)]^2}{t[1-r(t)]}+\frac{r^2(t)A^2(d,\rho)-1}{t^2}(t-\underline{t})]dF(t)\\&+\frac{n}{(2-d)^2}\int_{t^{**}}^{\bar{t}}[\frac{[1-r(t)A(d,\rho)]^2}{t[1-r(t)]}-\frac{r^2(t)A^2(d,\rho)-1}{t^2}(\bar{t}-t)]dF(t)\end{aligned}$$

(i) $\forall t\in[\underline{t},t^*]$, we denote $H(t,\tau(t))=V(t)+\frac{\partial V(t,t')}{\partial t}|_{t'=t}\frac{F(t)}{f(t)}=V(t,\tau(t))+\frac{\partial V(t,\tau(t'))}{\partial t}|_{t'=t}\frac{F(t)}{f(t)}$. $\forall t\in[t^{**},\bar{t}]$, we denote $G(t,\tau(t))=V(t)-\frac{\partial V(t,t')}{\partial t}|_{t'=t}\frac{1-F(t)}{f(t)}=V(t,\tau(t))-\frac{\partial V(t,\tau(t'))}{\partial t}|_{t'=t}\frac{1-F(t)}{f(t)}$. Thus to maximize $H(t,\tau(t))\ \forall t\in[\underline{t},t^*]$ and to maximize $G(t,\tau(t))\ \forall t\in[t^{**},\bar{t}]$ are pointwise maximization of $\Pi(t^*,\rho,\tau(.))$. By (ii) of the proof of Proposition 4, we have $\frac{\partial H(t,\tau(t))}{\partial\tau(t)}>0$. Thus $\tau^*(t)=\bar{\tau}\ \forall t\in[\underline{t},t^*]$ and the constraints $r(t)A(d,\rho)<1\ \forall t\in[\underline{t},t^*]$ are satisfied. $\forall t\in[t^{**},\bar{t}]$, we have the first order derivative of $G(t,\tau(t))$ with respect to $\tau(t)$ is

$$\frac{\partial H(t,\tau(t))}{\partial\tau(t)}=\frac{1}{(2-d)^2}[\frac{(r(t)A(d,\rho)-1)(r(t)A(d,\rho)-2A(d,\rho)-1)}{\tau^2(t)}+\frac{2r^3(t)A^2(d,\rho)}{t^3}(\bar{t}-t)]$$

Let $\frac{\partial H(t,\tau(t))}{\partial \tau(t)} = 0$ we have that $\forall t \in [t^{**}, \bar{t}]$,

$$[r(t)A(d,\rho)-1][r(t)A(d,\rho)-2A(d,\rho)-1] = 2r(t)A^2(d,\rho)[1-r(t)]^2(1-\frac{\bar{t}}{t})$$

Denote the solution of this equation as $r^*(t)$. Easy to check that the constraints $r^*(t)A(d,\rho) \geq 1$ $\forall t \in [t^{**}, \bar{t}]$ are satisfied. Also, $\tau^*(t)$ is bounded above by $t[A(d,\rho)-1]$. Thus $\tau^*(t) \leq \bar{\tau}$ $\forall t \in [t^{**}, \bar{t}]$.

(ii) Suppose the constraint $t^* \leq t^{**}$ is not binding. The first order derivative of $\Pi(t^*, t^{**}, \rho, \tau(.))$ with respect to $t^*$ is given by

$$\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial t^*} = \frac{n}{(2-d)^2}[\frac{[1-r(t^*)A(d,\rho)]^2}{t^*[1-r(t^*)]} + \frac{r^2(t^*)A^2(d,\rho)-1}{t^{*2}}(t^* - \underline{t})]$$

By $\underline{t} > \frac{t^*}{2}$, easy to check that this is positive. Thus $\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial t^*} > 0$. Also, the first order derivative of $\Pi(t^*, t^{**}, \rho, \tau(.))$ with respect to $t^{**}$ is given by

$$\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial t^{**}} = \frac{n}{(2-d)^2}[-\frac{[1-r(t^{**})A(d,\rho)]^2}{t^{**}[1-r(t^{**})]} - \frac{r^2(t^{**})A^2(d,\rho)-1}{t^{**2}}(\bar{t} - t^{**})]$$

Since $r(t^{**})A(d,\rho) \geq 1$, we have $\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial t^*} < 0$. Since $\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial t^*} > 0$ and $\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial t^{**}} < 0$, we must have the constraint $t^* \leq t^{**}$ is binding and $t^* = t^{**}$.

(iii) The first order derivative with respect to $\rho$ is given by

$$\begin{aligned}\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial \rho} = {} & \frac{n}{(2-d)^2}\frac{\partial A(d,\rho)}{\partial \rho}[\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]} + \frac{r^2(t)2A(d,\rho)}{t^2}(t-\underline{t})]dF(t) \\ & + \int_{t^{**}}^{\bar{t}}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]} - \frac{r^2(t)2A(d,\rho)}{t^2}(\bar{t}-t)]dF(t)]\end{aligned}$$

We have $\frac{\partial A(d,\rho)}{\partial \rho} = \frac{2d\int_{\{k|a(k)=1\}}(r(t)-1))dF(t)}{(2-d\rho\int_{\{k|a(k)=1\}}r(t)dF(t))^2} > 0$. By (i) we have that $\tau^*(t) = \bar{\tau}$ $\forall t \in [\underline{t}, t^*]$ thus $r(t) \to 0$ $\forall t \in [\underline{t}, t^*]$ and $\int_{\underline{t}}^{t^*}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]} + \frac{r^2(t)2A(d,\rho)}{t^2}(t-\underline{t})]dF(t) \to 0$. Also, $\forall t \in [t^{**}, \bar{t}]$, we have $r(t)A(d,\rho) \geq 1$ and thus $\int_{t^{**}}^{\bar{t}}[\frac{2(-r(t))[1-r(t)A(d,\rho)]}{t[1-r(t)]} - \frac{r^2(t)2A(d,\rho)}{t^2}(\bar{t}-t)]dF(t) < 0$. Thus $\frac{\partial \Pi(t^*, t^{**}, \rho, \tau(.))}{\partial \rho} < 0$. Since $r(t)$ is increasing in $t$, we must have $r(t)A(d,\rho) \geq 1$ is binding at $t = t^{**}$, i.e., $r^*(t^{**})A(d,\rho^*) = 1$. By (ii) we have $t^{**} = t^*$ and $r^*(t^*)A(d,\rho^*) = 1$.

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